Unit Title: Number and Shape Patterns

Lesson Adapted from: Grade 4: Unit 4.OA.C.5 Generate & Analyze Patterns, Maryland State Department of Education, July 30, 2013

NRS Level(s): Primarily Low to High Intermediate Basic Education (some Beginning Basic Education)	**Content Area(s):** Operations and Algebraic Thinking	**Length of Unit (e.g., hours, days):** 3-6 hours

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Unit Objective(s):

Students will be able to

- generate a number pattern that follows a given rule.
- generate a shape pattern that follows a given rule.
- generate both growing and repeating patterns that follow given rules.
- identify various features that they see in the patterns generated.
- determine a future element in a repeating or growing pattern based on the portion of the pattern visible.

Coherence:

Prerequisite or foundational content students need to succeed in the lesson:

- Fluency with multiplication facts

Description of how the content of the lesson is related to other content taught at the lesson's level: While this unit addresses number and shape patterns, this lesson could easily lead into other units on proportional relationships, graphing of proportional relationships, slope, and even unit rate.

Components of Rigor:

  x   Conceptual Understanding   x Procedural Skill and Fluency
  x   Application

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CCR Standard(s):

Primary Standard(s) (1-2 per lesson) :

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (4.OA.5)

Supporting Standard(s):

3.OA.1, 3.OA.2, 3.OA.3, 3.OA.4
4.OA.1
6.EE.6

ACES TIF Skill(s):

EF Skill 1a, 1b, 1e
CT Skill 2, Skill 3a, 3b

Northstar Digital Literacy Standard(s):

(Possible Extension Activity to bridge to graphing)
Microsoft Excel: 1. Open a workbook 5. Locate a cell 10. Enter data in a cell 18. Create a graph using data

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Common Career and Technical Core Standards:

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Standards for Mathematical Practice: *Only select the 2-4 practices that are <u>central</u> to the lesson*

  _x_MP 1: *Make sense of problems and persevere in solving them*   __MP 5: *Use appropriate tools strategically*

  __MP 2: *Reason abstractly and quantitatively*   _x_MP 6: *Attend to precision*

  __MP 3: *Construct viable arguments and critique the reasoning of others*   _x_MP 7: *Look for and make use of structure*

  __MP 4: *Model with mathematics*   __MP 8: *Look for and express regularity in repeated reasoning*

Level(s) of Knowing:

  _x_Intuitive: *Linking to what students already know*   _x_Abstract: *Writing with symbols and numbers*

  _x_Concrete: *Moving manipulatives*   _x_Application: *Applying to different situations*

  __Pictorial: *Drawing pictures*   _x_Communication: *Explaining concepts, process and/or solutions to others`1*

Materials:	**Potential Barriers to Student Learning:** (Process, Product, Content, Environment, Misconceptions)
Pre-test Multiplication Chart Resource 2A, 2B, 2C Resource 3A, 3B, 3C Post-test Selected GED questions from Steck-Vaughn prep book on patterns and functions Answer keys	Since this is a multi-level class, I anticipate the generating of rules being difficult for some. The abstract thinking could prove to be a barrier, so there are instructions for leveled groups in the lesson. *Volunteer assistance would be very helpful in order to manage this. **Adaptations and/or Accommodations:** <u>Struggling Learners</u> • Provide manipulatives for students to physically model the patterns • Physically create the model using desks and chairs (for shape pattern lesson) • Frontload specific vocabulary such as *level* and *sequence* • Allow students to work in small groups and/or partners <u>ELL</u> • Frontload specific vocabulary such as *level, sequence,* and *pattern* • Allow students to work in small groups and/or partners <u>Higher Level Students</u> • Have students extend patterns beyond classroom expectation • Students can design their own shape patterns and use a chart to record the patterns numerically • *Use the extra worksheets provided to extend learning to abstract rules (foundation for graphing lines) • Students can use Google Sheets or Microsoft Excel to create a graph of the information given in the tables

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Key Math Terms:	Academic Vocabulary and Additional Language Demands:
Number pattern, shape pattern, variable, rule	Level, sequence, pattern, PCA, increments, Chromebook, batch (of cookies), dozen

Lesson **ONE** Objective(s):	At the end of this lesson, students will be able to: • generate a number pattern that follows a given rule. • generate both growing and repeating patterns that follow given rules. • determine a future element in a repeating or growing pattern based on the portion of the pattern visible.
Assessing Mastery of the Objective(s): *(Indicate <u>when</u> and <u>how</u> assessment will occur during the lesson)*	By the end of this lesson, the students will be able to generate a number pattern and indicate the rule of the pattern as evidenced by completing problem 2C.
Student Learning Target(s):	"I can…" • continue a number pattern by filling in missing information • solve a word problem that involves a number pattern • describe a number pattern in words • describe a number pattern in terms of a variable (a general rule for any given number in the pattern)

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Introduction:	(Note: The day before the lesson, give pre-assessment. Collect and do not go over answers. Give it again at the end of the unit and see if results improved.)
	*Find and print a multiplication chart.
	Ask: Where have you seen patterns? Is anyone wearing a pattern? Does anyone see a pattern in the classroom?
	(As students share, have them explain each pattern explicitly.)
	Directions:
	Refer to multiplication chart – patterns
	Project multiplication chart AND distribute copies to students.
	Ask: What patterns do you see? (Give an example to illustrate what you are asking them to do.)
	Example: Rows & columns for 5 – all products end in 0 or 5
	Some others students might say:
	• Rows & columns for 10 – all products end in 0.
	• Diagonally from top left to bottom right – all perfect squares
	• A square number is a result of a number times itself (1 x 1, 2x2, etc.)
	• Look at where the square products fall on the table and identify where the next square number in the sequence will fall.
	Today we will be exploring more number patterns.

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Explanation & Modeling:	**Look at the students you have in class that day, and put them into two groups in the room: one that will be ready for delving into abstract thinking and one that will not. Have both groups positioned so that you have the option of doing some all-class work, but that when completing portions of the worksheets, you can have them work in leveled groups. Volunteer assistance will help manage this. Hand out 2A to the students. Read through the problem. Before continuing to solve, ask (aloud) yourself the following question about the TABLE: "What do I notice?" Share some things you notice: Look at the relationship between the two numbers in each row. (The week and the total saved for that week). We can see that if you multiply each week by 25, you get the total amount of money saved. So we can see that the rule is to <u>multiply the week times 25</u>. For now, stop at this descriptive explanation of the rule. Make sure to answer the question on the worksheet before moving on.
Guided Practice:	Hand out 2B to the students. Read through the problem. Before continuing to solve, ask students to individually write down an answer to the following question about the TABLE: What do you notice? Have students share responses. When students are sharing their responses, have them state whether the information they notice is extra information or if it will help them solve the problem. Then, as a class, fill in the answers to the questions. Once you fill in the descriptive rule (multiply the number of hours by 60), move on to the independent practice outlined below. (This is where students who are ready for the next step can later take it one step further to the abstract.)

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Independent Practice:	Part 2C provides students with an opportunity to practice what they have just learned. Have students try the problem on their own. When students have completed what they can, have students discuss their answers in pairs. Encourage student discussion as they work through the problem. As you walk around and listen to their conversations, challenge them to justify their reasoning to their neighbor. "<u>Why</u> are you doing that?" **See the handout below ("Guess My Rule"). The process for figuring out a multiplicative rule of a table of information is outlined in this handout. You can use a volunteer to lead the higher level students through this worksheet to make the connection to an abstract application of this concept. After going through this worksheet, the volunteer can return to the 2A and 2B examples and help students come up with y = 25x and y = 60x for the rules. Then, they can see if they can do 2C by themselves. As students are working, note which students are still struggling with the concept and might need more time/support. You might even make a list of students (use an attendance sheet) with "table", "first two questions", and "rule" as columns on the sheet. Put a check mark next to students who seem to be completing each type of question without much help from a teacher or each other. This will allow you to easily see if there is a majority of students who are either confused about or understand the questions. If students in the lower level group finish first (since they are not completing the additional worksheet), have them work in pairs to see if they can come up with their own number pattern. What numbers could they start with? By what factor do they want those numbers to change? OR They could come up with a scenario like the examples. (Pricing, running at a constant speed, etc.)

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| Student Reflection on Learning, Closure, Connection | Discuss solutions as a large group.

Have students silently review the "I can…" statements from the beginning. Ask them to write down on a piece of paper two different things: (1) what they are confident they could do on their own (2) what they still feel uncertain about. OR have students write down how we practiced each one (i.e. what lesson activities did we do to target each statement?) **Note: the fourth "I can…" statement is only for your higher leveled group.

For everyone, tell them that being able to decipher number patterns leads to graphing lines and finding out slope (essentially the rate at which a line rises or falls – by how much and in what direction it slants). You might even show a picture of a graphed line to say that the tables of information we just figured out can also be represented in this way. This is a future lesson! On the GED test, students will need to be able to identify slope from a graphed line or a table of information (or two points) (even an equation – but that direct application is for the higher level group at a later time). |

*Subsequent lessons in this unit: shape patterns and GED practice problems (Steck-Vaughn is a good resources for selecting appropriate problems from the "Patterns and Functions" section of the student book and workbook.) You may choose to do appropriate S-V GED problems involving number patterns before moving on to shape patterns.

**Possible subsequent units: graphing, slope, function tables (more basic input/output tables with all 4 operations (see link below), unit rates

***Possible extension activity for this lesson: To lead into graphing, students can create a graph using Google Sheets or Microsoft Excel. See Northstar Digital Literacy Standards in unit overview.

****For additional practice with identifying rules, you can use this resource:
https://www.eduplace.com/parents/hmcam/reviews/pdf/4/4hmmca-cr-09-03-rt.pdf
(white out a and b and replace it with x and y to avoid confusion at this point)

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Apply What You Have Learned

Solve each problem and show all your work in the space provided or on another sheet of paper.

1. Shannon makes $10.25 an hour as a PCA. She needs to earn $300 per week to pay for her weekly expenses. If her employer only hires people in weekly increments of 5 hours, what is the least number of hours Shannon would need to work to earn the money she needs?

Complete the table to find your answer.

| Hours Worked	Total Pay
5	$51.25
10	
15	
20	
25	
30	
35	

Additional questions:

Shannon worked 8 hours on Monday, 4 hours on Tuesday, and 5 hours on Wednesday.

How much money will she make over these three days of work?

_____

How many more hours does she need to work before Saturday (the end of the week) so she makes at least $300?

_____

What rule can we use to describe this data?

_____

2. Look at the pattern of blue triangles below.
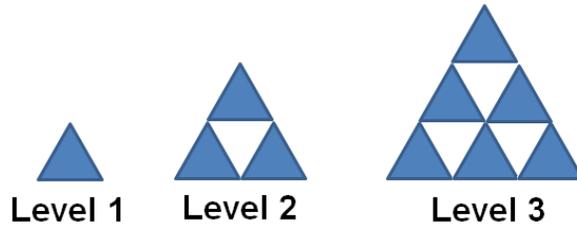
How many blue triangles are in each figure? _____

Think about the pattern you see. How many blue triangles will be in the 5th level? _____

3. Amin represented the pattern above with the numbers 1, 4, 9.

Explain what pattern Amin is seeing. _____

_____

How many triangles would be in the 5th level, according to Amin? _____

Using a Chart to Reason about Numeric Patterns

LEARNING HOW TO USE A CHART TO REASON ABOUT NUMERIC PATTERNS

Eli makes $560 per week as a construction worker. Eli plans to save $25 per week so he can buy a new Chromebook that costs $283.

We can use a chart to represent the total amount Eli is able to save each week.

Week	Total $ Saved
1	$25
2	$50
3	$75
4	$100

1. How many weeks will it take to save the amount he needs? _____

2. What rule can we use to describe this data?

Using a Chart to Reason about Numeric Patterns

LEARNING HOW TO USE A CHART TO REASON ABOUT NUMERIC PATTERNS

Ed is planning to drive from Minneapolis to Chicago, a distance of about 408 miles. Assume the average speed limit is 60 mph.

We can use a chart to represent the total amount of miles Ed will travel each hour. (Do not factor in bathroom breaks and rest stops.)

Hours	Total Miles
1	60
2	

1. How many miles will Ed have traveled after 3 hours?

   _____

2. How many hours will it take Ed to complete the trip?

   _____

3. What rule can we use to describe this data?

   _____

Using a Chart to Reason about Numeric Patterns

PRACTICE USING A CHART TO REASON ABOUT NUMERIC PATTERNS

Shalisa is making cookies for her friend's baby shower. She agreed to make 5 dozen cookies. She can fit 8 cookies on each cookie sheet. She is wondering how many batches of cookies she will have to make.

Use the chart to help solve the problem.

Number of cookie sheets	Total number of cookies
1	

How many batches does she have to make? _____

Will she have any cookies left over? _____

What rule can we use to describe this data? _____

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Apply What You Have Learned

Solve each problem and show all your work in the space provided or on another sheet of paper.

1. Shannon makes $10.25 an hour as a PCA. She needs to earn $300 per week to pay for her weekly expenses. If her employer only hires people in weekly increments of 5 hours, what is the least number of hours Shannon would need to work to earn the money she needs? _____30 hours_____

Complete the table to find your answer.

Hours Worked	Total Pay
5	$51.25
10	$102.50
15	$153.75
20	$205.00
25	$256.25
30	$307.50
35	$358.75

Additional questions:

Shannon worked 8 hours on Monday, 4 hours on Tuesday, and 5 hours on Wednesday.

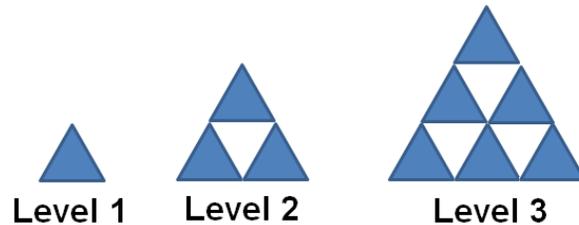How much money will she make over these three days of work?

_____$174.25_____

How many more hours does she need to work before Saturday (the end of the week) so she makes at least $300?

_____13 hours_____

What rule can we use to describe this data? ____multiply the hours worked by 10.25; for higher level learners: y =10.25x_____

2. Look at the pattern of blue triangles below.

Level 1   Level 2   Level 3

How many blue triangles are in each figure? _____1, 3, 6_____

Think about the pattern you see. How many blue triangles will be in the 5th level? __15 (The number of blue triangles is increasing each time: from the first level to the second level by 2, from the second to the third by 3, from the third to the fourth by 4, from the fourth to the fifth by 5.)_____

3. Amin represented the pattern above with the numbers 1, 4, 9.

Explain what pattern Amin is seeing. ___He is seeing the total number of triangles from one level to the next. Between the first level and the second level is 3, between the second level and the third level is 5. The pattern is going up by odd numbers. ____

How many triangles would be in the 5th level, according to Amin? _25 (1, 4, 9, 16, 25)_____

Using a Chart to Reason about Numeric Patterns

LEARNING HOW TO USE A CHART TO REASON ABOUT NUMERIC PATTERNS

Eli makes $560 per week as a construction worker. Eli plans to save $25 per week so he can buy a new Chromebook that costs $283.

We can use a chart to represent the total amount Eli is able to save each week.

Week	Total $ Saved
1	$25
2	$50
3	$75
4	$100
5	$125
6	$150
7	$175
8	$200
9	$225
10	$250
11	$275
12	$300

1. How many weeks will it take to save the amount he

     needs? _____12 weeks_____

2. What rule can we use to describe this data?

    __multiply the number of weeks by 25_____

    (For higher level group: y = 25x)

Using a Chart to Reason about Numeric Patterns

LEARNING HOW TO USE A CHART TO REASON ABOUT NUMERIC PATTERNS

Ed is planning to drive from Minneapolis to Chicago, a distance of about 408 miles. Assume the average speed limit is 60 mph.

We can use a chart to represent the total amount of miles Ed will travel each hour. (Do not factor in bathroom breaks and rest stops.)

Hours	Total Miles
1	60
2	120
3	180
4	240
5	300
6	360
7	420
8	480

1. How many miles will Ed have traveled after 3 hours? _____180 miles_____

2. How many hours will it take Ed to complete the trip? _____between 6 and 7 hours (advanced: 408/60 = 6.8 hours or 6 hours and 48 minutes)_____

3. What rule can we use to describe this data? ____multiply the number of hours by 60; for higher level groups: y = 60x (additional note: distance equals rate time time!)___

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Using a Chart to Reason about Numeric Patterns

PRACTICE USING A CHART TO REASON ABOUT NUMERIC PATTERNS

Shalisa is making cookies for her friend's baby shower. She agreed to make 5 dozen cookies. She can fit 8 cookies on each cookie sheet. She is wondering how many batches of cookies she will have to make.

Use the chart to help solve the problem.

Number of cookie sheets	Total number of cookies
1	8
2	16
3	24
4	32
5	40
6	48
7	56
8	64

How many batches does she have to make? _____8 batches_____

Will she have any cookies left over? _____yes, 4 cookies_____

What rule can we use to describe this data? _____multiply the number of cookie sheets by 8;

for higher level students: $y = 8x$___

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Guess My Rule
*Routine taken from Elizabeth Miller, Adult Options in Education

1. Figure out the relationship between the inputs and the outputs, and then fill in the rest of the table.

x (input)	y (output)
3	9
4	12
5	15
7	
9	
10	

2. Complete the sentence below:

   The _____ multiplied by _____ equals the _____.

3. Write the rule in an equation. Fill in the missing information. _____ = y

Guess My Rule Answer Key
*Routine taken from Elizabeth Miller, Adult Options in Education

1. Figure out the relationship between the inputs and the outputs, and then fill in the rest of the table.

x (input)	y (output)
3	9
4	12
5	15
7	21
9	27
10	30

2. Complete the sentence below:

   The _____input_____ multiplied by __3___ equals the _____output_____ .

3. Write the rule in an equation. Fill in the missing information. _____3x_____ = y