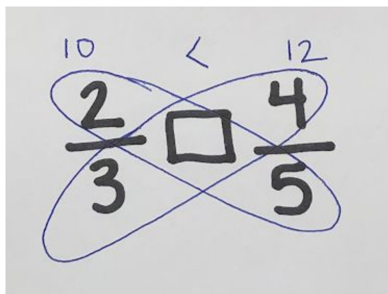
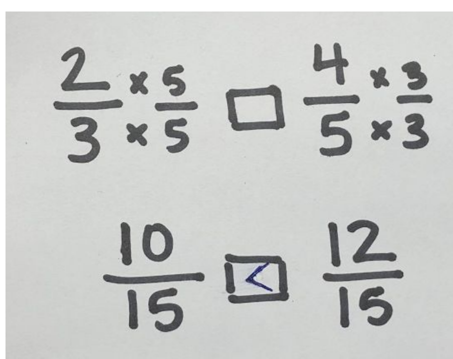
Why does cross-multiplication work when comparing fractions?

Look at the fractions being compared below. To determine which is the greater fraction, we can cross-multiply. When we cross-multiply these fractions, we get 2 x 5 = 10 and 3 x 4 = 12. Let's write the 10 above the first fraction and the 12 above the second fraction. Therefore, because 10 is less than 12, ⅔ is less than ⅘.



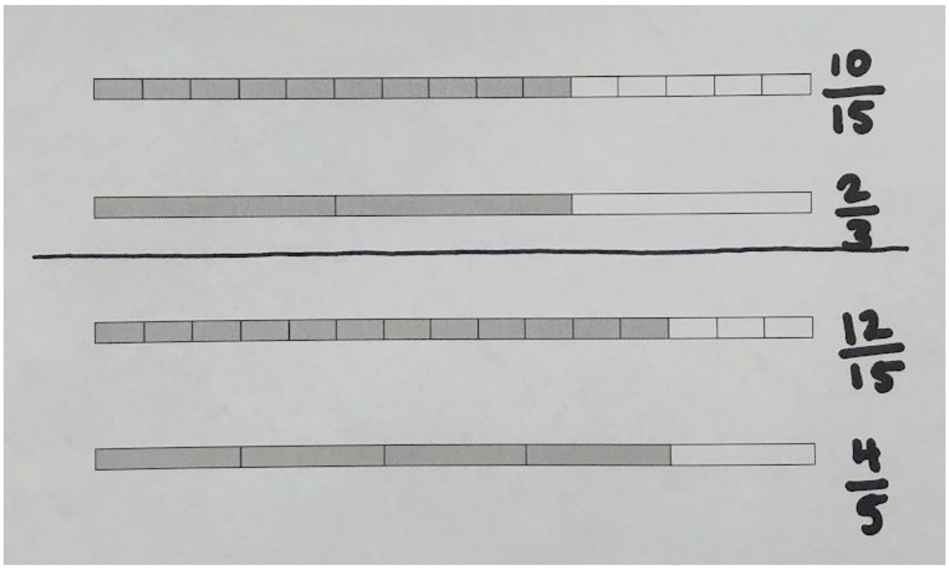
But wait...what is actually happening? Why does this work when comparing fractions that are referring to the same whole? Let's look at several different ways to think about this!

1) **Finding common denominators.** First, let's find a common denominator and then write equivalent fractions.

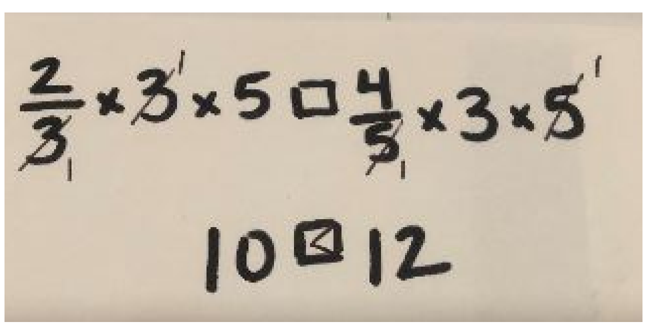


Notice how the numerators of the equivalent fractions are 10 and 12, respectively. Do these numbers look familiar? Essentially, by **cross-multiplying**, only the numerators of equivalent fractions are recorded. By comparing fractions using cross-multiplication, we lose the concept of finding equivalent fractions, which is *why* cross-multiplication works.

2) **Proportional Reasoning.** Similarly, we can also see where we get ‘10’ and ‘12’ by looking at these equivalent fractions using bar models.



3) **Property of Equality.** Let’s look at the Multiplication Property of Equality. This property states that if we multiply both sides of an equation or inequality by the same number, the values of each side remain equal. With our example of ⅔ and ⅘, if we multiply both sides of the inequality by 3 *and* by 5, we are maintaining the same value of each number. Essentially, when manipulating the numbers this way, it allows us to more easily see the comparative values.

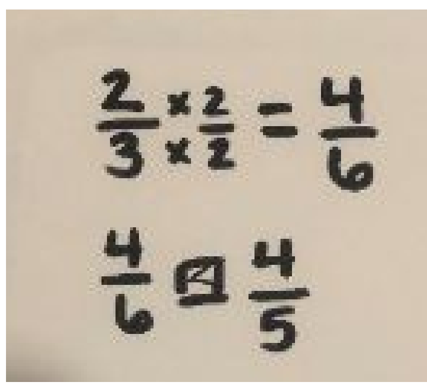


4) **Benchmark Fractions.** Guiding students to reason about the sizes of common fractions can help students compare values without even calculating! These

fractions can be used as benchmarks. For instance, $\frac{1}{2}$ is a benchmark fraction because it is easy to find half of a number. If a student wanted to compare $\frac{1}{3}$ and $\frac{3}{4}$ of some whole, that student could reason through what *half* of the whole would be to compare with what amount they have: Since half of 3 is 1.5, then $\frac{1}{3}$ is less than half of the whole. Comparatively, since half of 4 is 2, then $\frac{3}{4}$ is greater than half of the whole. Using the benchmark $\frac{1}{2}$, we know that $\frac{1}{3}$ is less than $\frac{3}{4}$.

Using a similar line of reasoning, let's look at $\frac{2}{3}$ and $\frac{4}{5}$ again. Both fractions are greater than $\frac{1}{2}$, so we need another benchmark. What if we look at how close these fractions are to 1 whole? Both fractions are only missing one piece in order to make the whole, BUT are those pieces the same size? $\frac{2}{3}$ is missing $\frac{1}{3}$, and $\frac{4}{5}$ is missing $\frac{1}{5}$. Since $\frac{1}{5}$ is a "smaller piece" than $\frac{1}{3}$, then $\frac{4}{5}$ is actually closer to 1. Therefore, $\frac{4}{5}$ is greater than $\frac{2}{3}$.

5) **Finding common numerators**. What if we found common numerators instead of common denominators? We could then reason about the size of the pieces and about how many we have.



Because sixths are smaller pieces than fifths, having 4 smaller pieces (4/6 or $\frac{2}{3}$) is less than having 4 larger pieces ($\frac{4}{5}$). Therefore, $\frac{2}{3}$ is less than $\frac{4}{5}$.

Instead of teaching students to memorize the method of cross-multiplication, we can use some of these ideas to discuss what is happening conceptually!